

# Neutrino Oscillations as a Lepton-Flavor-Violating Interaction

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## Abstract

To describe neutrino oscillations in the quantum mechanics sense, we propose to use an off-diagonal neutrino-Higgs (mass) interaction, as discussed recently in a family gauge theory. This extra orthogonal  $SU_f(3)$  family gauge theory may help us to resolve a few outstanding puzzles - the question of why there are only three generations, the question of why the masses of neutrinos are so tiny, and the question of why the dark-matter world is so huge (25%) as compared to the visible ordinary-matter world (5%).

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*Why are neutrinos so interesting?*

Neutrinos have masses, the tiny masses far below the range of the masses of the quarks and charged leptons. Because of the masses, neutrinos oscillate among themselves, giving rise to a lepton-flavor violation (LFV). Neutrino masses and neutrino oscillations may be regarded as one of the most important experimental facts over the last thirty years [1].

In fact, certain LFV processes such as  $\mu \rightarrow e + \gamma$  [1] and  $\mu + A \rightarrow A^* + e$  are closely related to the most cited picture of neutrino oscillations so far [1]. In this note, I wish to point out that the cross-generation or off-diagonal neutrino-Higgs interaction may serve as the detailed mechanism of neutrino oscillations, with some vacuum expectation value of the new Higgs field(s).

We have the outstanding question why there are three generations in the minimal Standard Model and, yet, another outstanding puzzle that the dark-matter world is about five times the visible ordinary-matter world (the latter described by the minimal Standard Model). Besides the role in the minimal Standard Model, neutrinos may be able to tell us something - an orthogonal family gauge theory in the dark-matter world which our neutrinos are capable of talking to (or interacting with).

Indeed, there is room left for something very interesting. Remember that the right-handed neutrinos never enter in the construction of the minimal Standard Model [2]. The message that the right-handed neutrinos seem to be "unwanted" could be telling us something. Now, the fact that neutrinos have tiny masses suggests that "more naturally" they would be four-component Dirac particles, and unlikely to be the two-component Majorana particles.

The room left for the right-handed neutrinos is that they are "unwanted" in the minimal Standard Model and that they could form some multiplet(s) under a new (dark-matter)

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gauge group besides the minimal Standard Model. We have some candidate from the symmetries - the family symmetry that there are three generations in the building blocks of (ordinary) matter, and so far only three. The puzzle so well-known that we no longer question ourselves why or why not! We have seen this fact, but we don't know why - let's speculate that it could be the story associated with the dark-matter world.

It arises naturally the so-called family gauge theory [3]. Note that the right-handed neutrinos do not appear in the minimal Standard Model. So, we could make a massive  $SU_f(3)$  gauge theory completely independent of the minimal Standard Model, including the particle content. We could treat  $(\nu_{\tau R}, \nu_{\mu R}, \nu_{e R})$  as a triplet under this  $SU_f(3)$  - so to give rise to a family gauge theory. Because the anomaly do not hurt in this case, we could drop the right-handed labels from the neutrinos. This completes the derivation of the family gauge theory [3]. The  $SU_f(3)$  is by definition the massive gauge theory - all the involved particles, except the neutrinos, are massive dark-matter particles.

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We treat the neutrino triplet  $(\nu_\tau, \nu_\mu, \nu_e) (\equiv \Psi)$  as the  $SU_f(3)$  triplet. Since the  $SU_f(3)$  family gauge theory is a massive gauge theory, we need two complex scalar triplets  $(\Phi_+, \Phi_-)$  to make all eight gauge bosons massive. To make the mass term, we have option to write

$$\eta \cdot \frac{1}{2} \cdot (i\bar{\Psi} \times (\Phi_+ + \epsilon\Phi_-) \cdot \Psi + h.c.), \quad (1)$$

which is an off-diagonal matrix (in  $SU_f(3)$ ). That is,  $\nu_e$  would transform into  $\nu_\mu$  or into  $\nu_\tau$ ,  $\nu_\mu$  would into  $\nu_\tau$  or  $\nu_e$ , and so on. This is interesting in view of neutrino oscillations, since it could be regarded as the underlying interaction (mechanism) for neutrino oscillations (which we are talking about [1]). An oscillation occurs in a way similar to the decay by way of creating a new species plus the vacuum expectation value (or, changing the vacuum). In quantum mechanics, this may be so far the only way how an oscillation can occur.

To illustrate the point further, we calculate the golden lepton-flavor-violating decay  $\mu \rightarrow e + \gamma$  as the celebrated example. We show in Figures 1(a), 1(b), and 1(c) three leading basic Feynman diagrams. Here the conversion of  $\nu_\mu$  into  $\nu_e$  is marked by a cross sign and it is a term from our off-diagonal interaction given above with the Higgs vacuum expectation values  $u_+$  and  $u_-$ . Here the Higgs masses are assumed to be very large, i.e., greater than a few  $TeV$ , as in  $SU_f(3)$ . The only small number (coupling) is  $\eta$ , explaining the tiny masses of neutrinos.

Using Feynman rules from Wu and Hwang [2], we write, for Fig. 1(a),

$$\begin{aligned} & \frac{1}{(2\pi)^4} \int d^4q \cdot \bar{u}(p', s') \cdot \quad i \cdot \frac{1}{2\sqrt{2}} \frac{e}{\sin\theta_W} \cdot i\gamma_\lambda (1 + \gamma_5) \\ & \cdot \frac{1}{i} \frac{m_2 - i\gamma \cdot q}{m_2^2 + q^2 - i\epsilon} \cdot i \cdot i\eta(-)(u_+ + \epsilon u_-) \cdot \frac{1}{i} \frac{m_1 - i\gamma \cdot q}{m_1^2 + q^2 - i\epsilon} \\ & \cdot i \cdot \frac{1}{2\sqrt{2}} \frac{e}{\sin\theta_W} \cdot \quad i\gamma_{\lambda'} (1 + \gamma_5) \cdot u(p, s) \\ & \cdot \frac{1}{i} \frac{\delta_{\lambda'\mu}}{M_W^2 + (p-q)^2 - i\epsilon} \cdot \frac{\epsilon_\sigma(k)}{\sqrt{2}k_0} \cdot \quad \Delta_{\sigma\mu\nu} \cdot \frac{1}{i} \frac{\delta_{\nu\lambda}}{M_W^2 + (p-q-k)^2 - i\epsilon}, \end{aligned}$$

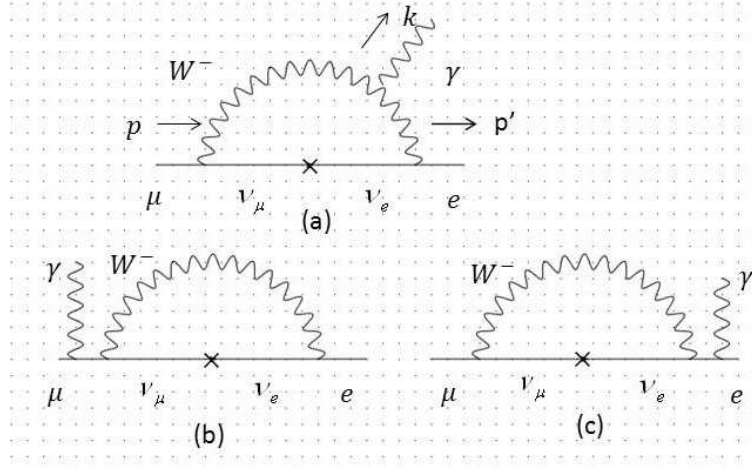


Figure 1: The leading diagrams for  $\mu \rightarrow e + \gamma$ .

with  $\Delta_{\sigma\mu\nu} = (-ie)\{\delta_{\mu\nu}(-k - p - q)_\sigma + \delta_{\nu\sigma}(p - q + p - q - k)_\mu + \delta_{\sigma\mu}(-p + q + k + k)_\nu\}$ .  
On the other hand, Feynman rules yield, for Fig. 1(b),

$$\begin{aligned} & \frac{1}{(2\pi)^4} \int d^4q \cdot \bar{u}(p', s') \cdot i \cdot \frac{1}{2\sqrt{2}} \frac{e}{\sin\theta_W} \cdot i\gamma_\lambda(1 + \gamma_5) \\ & \cdot \frac{1}{i} \frac{m_2 - i\gamma \cdot q}{m_2^2 + q^2 - i\epsilon} \cdot i \cdot i\eta(-)(u_+ + \epsilon u_-) \cdot \frac{1}{i} \frac{m_1 - i\gamma \cdot q}{m_1^2 + q^2 - i\epsilon} \\ & \cdot i \cdot \frac{1}{2\sqrt{2}} \frac{e}{\sin\theta_W} \cdot i\gamma_{\lambda'}(1 + \gamma_5) \cdot \\ & \cdot \frac{1}{i} \frac{\delta_{\lambda\lambda'}}{M_W^2 + (p' - q)^2 - i\epsilon} \cdot \frac{1}{i} \frac{m_\mu - 1\gamma \cdot p'}{m_\mu^2 + p'^2 - i\epsilon} \cdot i(-i)e\gamma_\sigma \cdot \frac{\epsilon(k)}{\sqrt{2k_0}} \cdot u(p, s), \end{aligned}$$

and a similar result for Fig. 1(c).

The four-dimensional integrations can be carried out, via the dimensional integration formulae (e.g. Ch. 10, Wu/Hwang [2]), especially if we drop the small masses compared to the W-boson mass  $M_W$  in the denominator. In this way, we obtain

$$\begin{aligned} T_a = \frac{G_F}{\sqrt{2}} \cdot \eta(u_+ + \epsilon u_-) \cdot (m_1 + m_2) \cdot (+2i) \frac{e}{(4\pi)^2} \\ \cdot \bar{u}(p', s') \frac{\gamma \cdot \epsilon}{\sqrt{2k_0}} (1 + \gamma_5) u(p, s). \end{aligned}$$

It is interesting to note that the wave-function renormalization, as shown by Figs. 1(b) and 1(c), yields

$$\begin{aligned} T_{b+c} = \frac{G_F}{\sqrt{2}} \cdot \eta(u_+ \epsilon u_-) (m_1 + m_2) \cdot (+2i) \frac{e}{(4\pi)^2} \cdot \left\{ \frac{p'^2}{m_\mu^2 + p'^2} + \frac{p^2}{m_e^2 + p^2} \right\} \\ \cdot \bar{u}(p', s') \frac{\gamma \cdot \epsilon}{\sqrt{2k_0}} (1 + \gamma_5) u(p, s), \end{aligned}$$

noting that  $p^2 = -m_\mu^2$  and  $p'^2 = -m_e^2$  would make the contribution from Figs. 1(b) and 1(c) to be the same as that from Fig. 1(a).

In a normal treatment, one ignores the wave-function renormalization diagrams 1(b) and 1(c) in the treatment of the decays  $\mu \rightarrow e + \gamma$ ,  $\mu \rightarrow 3e$ , and  $\mu + A \rightarrow e + A^*$ .

Comparing this to the dominant mode  $\mu \rightarrow e \bar{\nu}_e \nu_\mu$  [2], we obtain the branching ratio easily. Even though the decay rate for  $\mu \rightarrow e + \gamma$ , as obtained here, would be of the order  $(m_{\text{neutrino}}/m_\mu)^4$ , which is rather small, it is much larger than order  $(m^2/M_W m_\mu)^2$  as quoted in the literature.

The off-diagonal mass matrix would be modified by the self-energy diagram since the neutrinos form a triplet under  $SU_f(3)$ . It is presumed that these self-energy diagrams, after the ultraviolet divergences get subtracted, lead to masses of the right order. If the off-diagonal mass matrix is diagonalized alone, the three roots would be two negative and one positive, adding up to zero. This seems like one ordering in the masses of neutrinos - one up and two downs.

Besides the golden decay  $\mu \rightarrow e + \gamma$  (much too small) and neutrino oscillations (already observed), violation of the  $\tau - \mu - e$  universality is also expected and might be observed. As the baryon asymmetry is sometime attributed to the lepton-antilepton asymmetry, the current scenario for neutrino oscillations [1] seems to be inadequate. If we take the hints from neutrinos rather seriously, there are so much to discover, even though the minimal Standard Model for the ordinary-matter world remains to be intact.

We believe that, in the dark-matter world, the dark-matter particles are also species in the extended Standard Model. Most of reactions happening among dark-matter particles, even involving neutrinos, cannot be detected in the ordinary-matter world. It is clear that the minimum extended Standard Model would be the extended Standard Model to be based on the group  $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3) \times SU_R(2)$ , provided that why there are three generations of fermions could be explained and the missing right-handed sector would eventually be coming back [4].

In a slightly different context [5], I propose to work with two working rules: "Dirac similarity principle", based on eighty years of experience, and "minimum Higgs hypothesis", from the last forty years of experience. Using these two working rules, the extended model mentioned above becomes rather unique - so, it is so much easier to check it against the experiments.

We would be curious about how the dark-matter world looks like, though it is difficult to verify experimentally. The first question would be: The dark-matter world, 25 % of the current Universe (in comparison, only 5 % in the ordinary matter), would clusterize to form the dark-matter galaxies, maybe even before the ordinary-matter galaxies. The dark-matter galaxies would then play the hosts of (visible) ordinary-matter galaxies, like our own galaxy, the Milky Way. Note that a dark-matter galaxy is by our definition a galaxy that does not possess any ordinary strong and electromagnetic interactions (with our visible ordinary-matter world). This fundamental question deserves some thoughts, for the structural formation of our Universe.

Of course, we should remind ourselves that, in our ordinary-matter world, those quarks can aggregate in no time, to hadrons, including nuclei, and the electrons serve to neutralize the charges also in no time. Then atoms, molecules, complex molecules, and so on. These serve as the seeds for the clusters, and then stars, and then galaxies, maybe in a time span of 1 *Gyr* (i.e., the age of our young Universe). The aggregation caused by strong and electromagnetic forces is fast enough to help giving rise to galaxies in a time span of 1 *Gyr*.

On the other hand, the seeded clusterings might proceed with abundance of extra-heavy dark-matter particles such as familons and family Higgs, all greater than a few  $TeV$  and with relatively long lifetimes (owing to very limited decay channels). So, further simulations on galactic formation and evolution may yield clues on our problem.

Finally, coming back to the fronts of particle physics, neutrinos, especially the right-handed neutrinos, might couple to the dark-matter particles. Any further investigation along this direction would be of utmost importance. It may shed light on the nature of the dark-matter world.

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